

SOLAR SYSTEM DYNAMICS

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I. Rotational Dynamics of Irregularly Shaped Satellites

All irregularly shaped natural satellites tumbled chaotically at the point of capture into synchronous rotation.

The basic mechanism governing the tidal evolution of the spins and obliquities has been well understood since the work of George Darwin (1879). If the spin angular velocity is large the obliquity is driven to a equilibrium value between 0 and 90°. As the spin is slowed by tidal friction the equilibrium obliquity decreases. If the orbit is fixed, the equilibrium obliquity goes to zero as the spin angular velocity approaches twice the mean orbital motion. For smaller angular velocities the obliquity is zero. Spin-orbit resonances are encountered as the spin rate is reduced and the rotation may or may not be captured in a spin-orbit resonance with calculable probability (Goldreich and Peale, 1966). If the non-synchronous spin-orbit resonances are avoided, the spin continues to decrease until synchronous rotation is established.

A dramatic exception to the regular tidal evolution envisioned in the standard scenario is provided by the chaotic tumbling of Saturn's satellite Hyperion (Wisdom, Peale, and Mignard, 1984). The chaotic tumbling of Hyperion is a consequence of its highly aspherical shape, to some extent the large orbital eccentricity ($e \approx 0.1$), and the fact that the spin rate of Hyperion has been brought by tidal friction to be nearly synchronous with the orbital mean motion.

For other irregularly shaped satellites this picture of steady evolution to synchronous rotation must also be revised. All resonances are surrounded by a chaotic zone (see Chirikov, 1979). The width of the chaotic zone surrounding the synchronous spin-orbit state may be estimated. It turns out to be linearly proportional to the orbital eccentricity, but exponentially dependent on the parameter which measures the out of roundness of the satellite. Thus the chaotic zone surrounding the synchronous state may be significant for an irregularly shaped satellite even if the eccentricity is not as large as that of Hyperion. This prediction has been verified numerically. A remarkable fact is that this chaotic zone seems to always be attitude unstable just as it is for Hyperion. Even Deimos which has an anomalously low eccentricity of 0.0005 has a non-negligible chaotic zone which is attitude unstable. In all cases the timescale for the spin axis to fall away from the orbit normal is only a few orbit periods; the attitude instability is very strong. Thus all the irregularly shaped satellites in the solar system, regardless of their orbital eccentricity, tumbled chaotically at the point of capture into synchronous rotation.

Dissipation within a chaotically tumbling satellite is significantly greater than in a synchronously rotating satellite. If a secular change in the angular momentum of the satellite can be ruled out, then the dissipation within the satellite leads to a decrease in the orbital eccentricity. It is plausible, but not yet proven, that tidal friction will not give a secular change in the angular momentum of a chaotically tumbling satellite because, speaking qualitatively, the motion is so irregular the tides do not know which way to push. Once this is rigorously justified then the anomalously low eccentricity of Deimos can be explained as resulting from the dissipation in Deimos during the chaotic tumbling phase. In any case, the chaotic tumbling phase must be taken into account in the orbital histories of the natural satellites.

This work has been submitted to Icarus.

II. Origin of the Kirkwood Gaps

We have used the Digital Orrery (Applegate, et al. 1985) to systematically explore the phase space of the elliptic restricted three body problem near the principal commensurabilities ($2/1$, $5/2$, $3/1$, and $3/2$). The results for the $3/1$ commensurability are in close agreement with those found earlier with the algebraic mapping method (Wisdom 1981, 1983). Large chaotic zones are associated with the $3/1$, $2/1$ and $5/2$ resonances, where there are gaps in the distribution of asteroids. The region near the $3/2$ resonance, where the Hilda group of asteroids is located, is largely devoid of chaotic behavior. Thus there is a qualitative agreement between the character of the motion and the distribution of asteroids.

The detailed comparisons (using the representative plane method of Wisdom, 1983) between the distribution of asteroids and the chaotic zones do not show perfect agreement. While the boundaries on the large semimajor axis side are in good agreement, there is in each case a small region at fairly low eccentricity on the small semimajor axis side of the gaps where there are no asteroids and also no chaotic behavior. Unfortunately, the interpretation of this result is not straightforward. There are three possibilities: (1) The planar elliptic approximation is not a sufficiently accurate representation of the motion near the principal commensurabilities. (2) The void on the representative plane represents a sufficiently small phase space volume that asteroids were simply unlikely to be found there. (3) Cosmogonic mechanisms such as resonance sweeping (Henrard and Lemaître, 1983, and Torbett and Smoluchowski, 1980) have been operative. The first possibility has been examined by integrating several test particles in the field of the major planets with the Digital Orrery. Five test particles were started in the region in question and integrated for five million years each. Of the five test particles three turned out to be chaotic. This gives strong support to the conclusion that the discrepancy is primarily due to the use of the elliptic restricted approximation in the systematic exploration. Most likely a combination of the first two possibilities can fully account for the differences between the systematic exploration and the observed distribution, but resonance sweeping can not yet be ruled out. We intend to resolve this question soon.

The existence of chaotic zones at the principal commensurabilities does not by itself explain the existence of the Kirkwood gaps. We must also understand the mechanism whereby asteroids on chaotic orbits are removed. We have used the Digital Orrery to approach this problem as well. Again, five test particles were integrated in the field of the major planets for five million years each. Each one was started in the $2/1$ chaotic zone. Of these five, two reached eccentricities above 0.6, which makes them Mars crossing. In both cases they went to large eccentricity by a path that temporarily took them to high inclination ($i > 20^\circ$). (Recall that the integrations by Froeschlé and Scholl (1981) of Giffen's (1973) $2/1$ chaotic orbit seemed to show that there was a maximum eccentricity. Their integrations were performed in the planar approximation.) This is quite an interesting result. This is the first example in the solar system where a phenomenon seems to depend on the interconnectedness of the chaotic zone in many dimensions (the so-called "Arnold web", see Chirikov, 1979). Sweeping by Mars thus appears to be a plausible removal mechanism, though it seems likely that longer integrations will reveal Jupiter itself to be the culprit.

References

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